Philosophy of Mathematics

Daniel Waxman

This course investigates some of the deep and puzzling philosophical issues arising from mathematics. Some are metaphysical and semantic: What is mathematics about? Are there such things as mathematical objects (numbers, functions, sets, etc), and if so, do they exist in the same way as more familiar entities such as tables and chairs? Others are epistemological: How do we obtain mathematical knowledge? Does it arise from pure reason alone, or does the empirical world play a role? In the course of addressing such questions, we will examine a number of influential views, including logicism, constructivism, platonism, fictionalism, and structuralism.

For the first half of the course, the required text will be Velleman and George: *Philosophies of Mathematics*.

Week 1: **Introduction**
       Velleman and George: *Philosophies of Mathematics*, Ch. 1

Week 2: **Frege's Logicism**
       Frege: *Foundations of Arithmetic*, Selections
       Velleman and George, Ch. 2
       *Boolos: *Logic, Logic, and Logic*, Part II
       *Hale and Wright: *The Reason's Proper Study*

Week 3: **Russell, Russell's Paradox, and Set Theory**
       Russell: Letter to Frege
       Frege: Reply to Russell
       Velleman and George: Ch. 3
       *Russell: Mathematical Logic as Based on the Theory of Types
       *Boolos: The Iterative Conception of Set

Week 4: **Intuitionism**
       Heyting: The Intuitionist Foundations of Mathematics
       Velleman and George: Chs. 4 and 5
       *Brouwer: Mathematics, Science, and Language
       *Dummett: *Elements of Intuitionism*

Week 5: **Finitism and Hilbert's Program**
       Hilbert: On the Infinite
       Velleman and George: Ch. 6
       *Tait: Finitism

Week 6: **Mathematical Truth**
       Benacerraf: Mathematical Truth
       Tait: Truth and Proof

Week 7: **Platonism and Indispensibility**

Putnam: What is Mathematical Truth?
Colyvan: *The Indispensibility of Mathematics*, Chs. 1-3 (the rest is optional)
Maddy: Indispensibility and Practice

Week 8: **Nominalism**
Field: Science Without Numbers (selections)
Burgess: Why I am not a Nominalist
Colyvan: The Indispensibility of Mathematics, Ch. 4
*Shapiro: Conservativeness and Incompleteness

Week 9: **Structuralism**
Benacerraf: What Numbers Could Not Be
Parsons: The Structuralist View of Mathematical Objects
*MacBride: Structuralism Reconsidered

Week 10: **Pluralism**
Balaguer: Platonism and Anti-Platonism in Mathematics (selections)
Hamkins: The Set Theoretic Multiverse

Week 11: **Justifying the axioms**
Maddy: Believing the Axioms
Feferman: Does Mathematics Need New Axioms?